Illumination and Spillover Efficiency Calculations for Rectangular Reflectarray Antennas

By Michał Żebrowski

Reflectarray antennas can be an interesting alternative to classic array antennas.

Introduction

The reflectarray antenna was first introduced by Barry and Malech in their paper [1] from 1963. A reflectarray antenna consists of an illuminating antenna (feed) and a flat or curved reflector built of reflectarray elements (see Fig. 1).

Reflectarray elements retransmit signals from the feed and introduce such phase delay that a plane wave is formed in a given direction. Reflectarrays may be an interesting alternative to classic array antennas [2], because they do not contain a beamforming network in its classic form. Instead, the free space between the feed and reflector aperture acts as a beamforming network.

Lack of a traditional beamformer leads to increased reliability of the entire antenna, because the reflectarray elements are isolated from each other and decrease the total weight of antenna. In addition, when the reflector contains several thousand elements [3] there is no need to build an extensive signal divider, which would be very complicated in case of classic array antennas. However, it needs to be stressed that in reflectarray antennas, using free space as the beamforming network leads to a fundamental design problem: how to choose the proper mutual position between the feed and the reflector.

Another problem, connected with the former, is the choice of a correct radiation pattern of the feed. All these parameters affect the reflectarray efficiency, which should obviously be possibly high. In this paper formulas are derived which describe the relationship between parameters of a rectangular reflectarray and its antenna efficiency (characterized by two components: illumination efficiency and spillover efficiency). By plotting antenna efficiency versus certain antenna parameters, the designer can choose parameter values which maximize the reflectarray efficiency.

Definitions of Illumination and Spillover Efficiencies

The definitions of illumination and spillover efficiencies [3][4][5] utilize three power

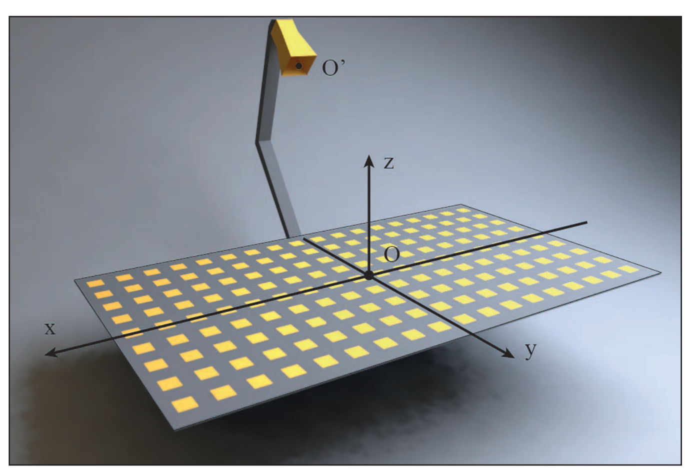
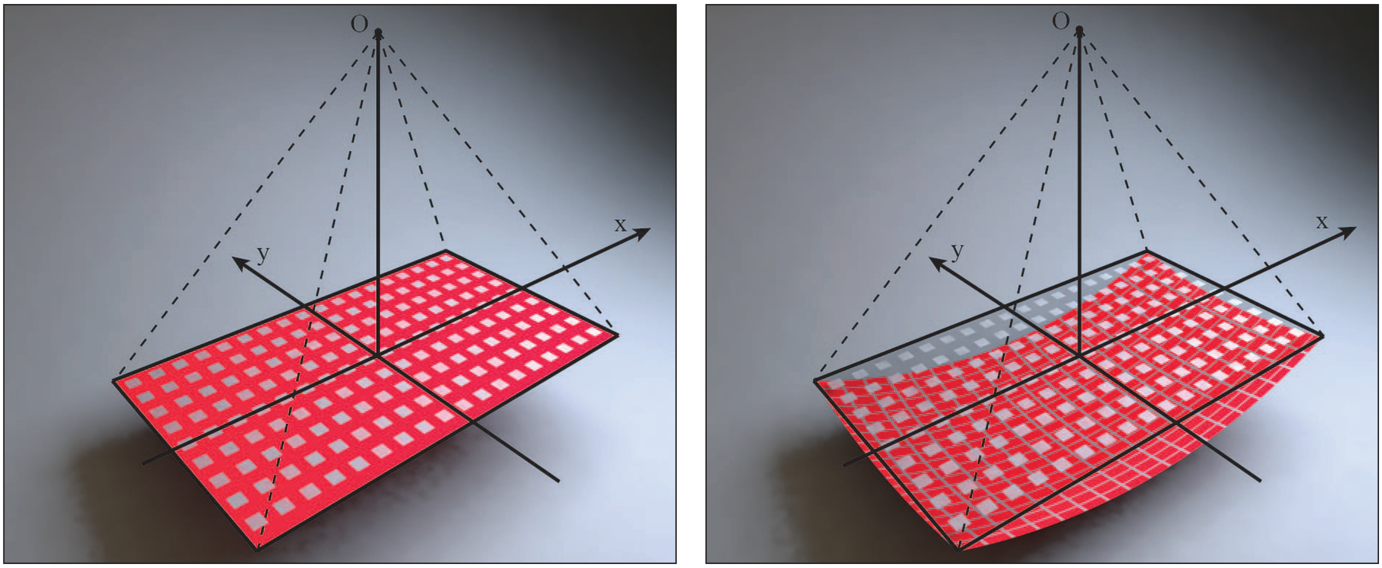


Figure 1 • Reflectarray schematic with reflectarray elements and feed marked in yellow.



(a) **(b)**

Figure 2. Integration areas for integral A (a) and B (b) marked with red color.

values, which will be represented by A, B and C. Power A will denote power received by reflectarray elements and radiated by the feed (Fig. 2a). Power B will represent power radiated by the feed into a section of space limited by the reflector (Fig. 2b). The last power C will indicate the whole power radiated by the feed into the front hemisphere.

Figure 2 shows areas which are used to calculate individual powers A, B and C. According to considerations from [5][6][7] definitions of these powers lead to definitions of illumination and spillover efficiencies. Illumination efficiency is defined as [7][8]:

$$\eta_{ill} = A/(s\cdot B) \qquad (1)$$

and spillover efficiency is defined as:

$$\eta_{spill} = B/C\,. \qquad (2)$$

Antenna efficiency definitions for circular reflectarrays have been published [8][9][10], but to the author's knowledge no formulas have been published so far, making it possible to calculate this efficiency for rectangular reflectarrays. Some authors [9][10] mention that appropriate formulas may be calculated by proper integrations. The authors of paper [7] presented a general method of illumination and spillover efficiency calculation, but they have utilized it only for a circular reflector. The current paper holds formulas that may be used to calculate illumination and spillover efficiencies for reflectarrays with rectangular reflectors. An effective numerical integration method of Simpson [5] is proposed and used for calculation of illumination and spillover efficiencies. It is also shown that under strict conditions, closed form formulas derived for circular reflectarrays can be applied to rectangular reflectarrays in the early stage of design.

Formulas for the Calculation of Illumination and Spillover Efficiencies for a Reflectarray with Rectangular Reflector

Let us consider a reflectarray with its feed placed normal to the reflector aperture. Let the feed radiate electromagnetic wave polarized linearly along 0y axis. The electric field intensity of the wave can be written using the $\cos^q$ model [12] in the following form:

$$\bar{E}_{feed}(\theta) \cong \frac{1}{r}\cdot\left[F_E(\theta)\sin\varphi\cdot\bar{i}_\theta + F_H(\theta)\cos\varphi\cdot\bar{i}_\varphi\right] \qquad (3)$$

where $F_E(\theta) = \cos^{q_E}(\theta)$, $F_H(\theta) = \cos^{q_H}(\theta)$ are feed radiation patterns in E field and H field planes, respectively.

Let us assume that the radiation pattern of a single reflectarray element is given by $F_{element}(\theta) = \cos(\theta)$, which means that the 3dB beamwidth of this pattern amounts to 90° in both principal planes. Power A, defined in paragraph 2, is determined by:

$$A = \left|\iint_S \bar{E}_{feed}\cdot\bar{i}_y\cdot E_{element}\cdot dS\right|^2 \qquad (4)$$

where S is the surface of the reflector aperture. In order to calculate this power only the 0y polarized component is used, because it was assumed, that reflectarray elements receive 0y polarized signals. The 0y component may be calculated from:

$$E_y = \bar{E}_{feed} \cdot \bar{i}_y = E_{feed,\theta} \cos\theta \sin\varphi + E_{feed,\varphi} \cos\varphi \quad (5)$$

For electric field intensity in (4) the 0y component takes the following form:

$$E_y \cong \frac{1}{r} F_E(\theta) \cos\theta \sin^2\varphi + \frac{1}{r} F_H(\theta) \cos^2\varphi \quad (6)$$

Based on the assumptions so far, one can calculate power B (see definition in paragraph 2) using the following formula:

$$B = \iint_{S'} \left| \bar{E}_{feed} \right|^2 dS' \quad (7)$$

where S' is the surface of the sphere, limited by the reflector aperture (see Fig. 2b). The last of the powers from paragraph 2, power C, can be calculated from:

$$C = \iint_{S''} \left| \bar{E}_{feed} \right|^2 dS'' \quad (8)$$

where S'' is the surface of the hemisphere in front of the feed. Formulas for powers A and B have an appropriate form for integration, provided that the reflector is of circular shape [8][10]. However, when rectangular reflector is concerned, it is better to define these powers using Cartesian coordinates, on the reflector plane. In order to do this, supplementary figures 3a and 3b will be used.

Using Figure 3, one can rewrite individual factors in the formulas for powers A and B, in Cartesian coordinates:

$$\cos(\theta) = \frac{R}{m}, \ \sin\varphi = \frac{y}{\sqrt{x^2+y^2}}, \ \cos\varphi = \frac{x}{\sqrt{x^2+y^2}}$$

$$\text{where } m = \sqrt{R^2+x^2+y^2} \quad (9)$$

Geometrical dependencies from Fig. 3 can also be used to calculate area elements dS and for integrals A and B. In case of integral A, the integration is carried out in Cartesian coordinates over a flat surface, hence $dS=1 \cdot dx \cdot dy$. As far as integral B is concerned the integration is carried out in Cartesian coordinates over a spherical surface. In the latter case area element dS' can be calculated using a parametric description of the spherical surface:

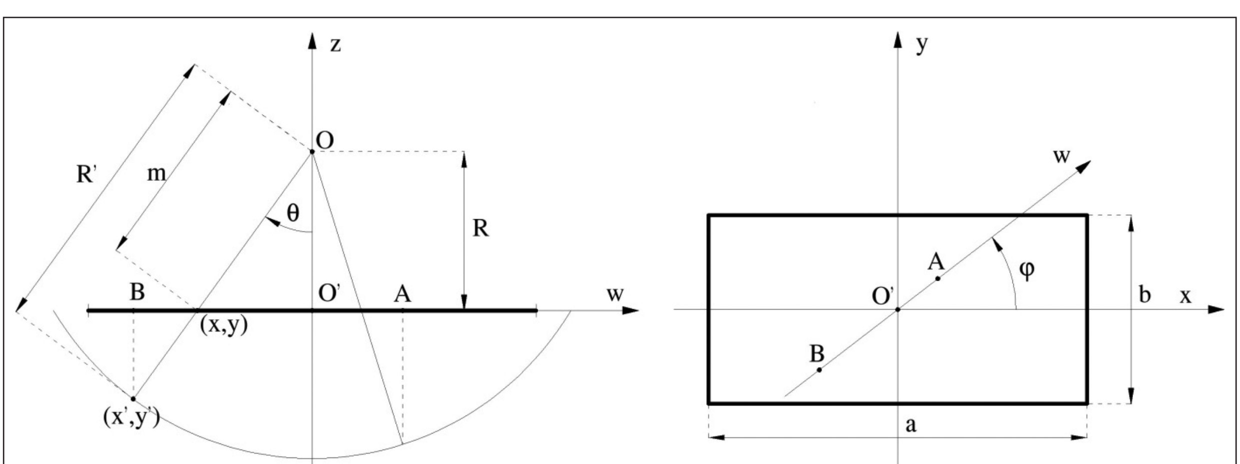
$$x'(x,y) = \frac{xR'}{m}$$
$$y'(x,y) = \frac{yR'}{m}$$
$$z'(x,y) = R \cdot \left(1 - \frac{R'}{m}\right), \quad (10)$$

$$\text{where } R' = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + R^2}$$

and m was defined in (9).

According to considerations in [12] area element dS'can be calculated using the Jacobian of the following transformation $(x,y) \rightarrow (x',y',z')$. The Jacobian can in turn be calculated from the following formula:

$$J' = \left\| \begin{matrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial x} & \frac{\partial z'}{\partial x} \\ \frac{\partial x'}{\partial y} & \frac{\partial y'}{\partial y} & \frac{\partial z'}{\partial y} \\ i_x & i_y & i_z \end{matrix} \right\| \quad (11)$$



(a) (b)

Figure 3 • Cross-sections of a flat, rectangular reflector with feed placed at O.

Formula (11), after some straightforward transformations, can be rewritten as:

$$J' = (R')^2 \cdot R \cdot m^{-3} \qquad (12)$$

Then area element dS' can be written as $dS' = J' \cdot dx \cdot dy$ and integral B as:

$$B = \int_{x=-a/2}^{a/2} \int_{y=-b/2}^{b/2} \left[\left(\frac{R}{\sqrt{R^2 + x^2 + y^2}} \right)^{2q_E} \cdot \frac{y^2}{x^2 + y^2} + \left(\frac{R}{\sqrt{R^2 + x^2 + y^2}} \right)^{2q_E} \cdot \frac{x^2}{x^2 + y^2} \right] \cdot \frac{R}{\left(\sqrt{R^2 + x^2 + y^2} \right)^3} dx \cdot dy \qquad (13)$$

Integral A takes the following form in Cartesian coordinates:

$$A = \int_{x=-a/2}^{a/2} \int_{y=-b/2}^{b/2} \left\{ \frac{1}{\sqrt{R^2 + x^2 + y^2}} \cdot \left[\left(\frac{R}{\sqrt{R^2 + x^2 + y^2}} \right)^{q_E+2} \cdot \frac{y^2}{x^2 + y^2} + \left(\frac{R}{\sqrt{R^2 + x^2 + y^2}} \right)^{q_H+1} \cdot \frac{x^2}{x^2 + y^2} \right] \right\} \cdot 1 \cdot dx \cdot dy \qquad (14)$$

Integral C is easily calculated in spherical coordinates, because of the shape of the integration surface and the form of electric field intensity (eq. 6):

$$C = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\cdot\pi} \frac{1}{r^2} \left[\cos^{2q_E}(\theta) \cdot \sin^2(\varphi) + \cos^{2q_H}(\theta) \cdot \cos^2(\varphi) \right] \cdot r^2 \sin\theta \cdot d\theta \cdot d\varphi \qquad (15)$$

After analytical integration (by substitution) one can obtain the following result:

$$C = \pi \cdot \left(\frac{1}{1 + 2 \cdot q_E} + \frac{1}{1 + 2 \cdot q_H} \right) \qquad (16)$$

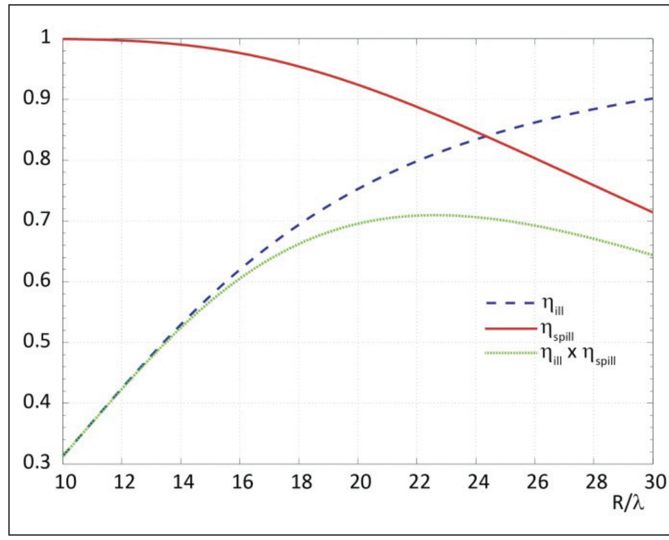
Integrals A and B given by (13) and (14) cannot be calculated analytically. That is why an effective numerical method—the Simpson method—was used to calculate them. This integration method is described in Appendix 1.

Examples of Usage of the Efficiency Formulas for Reflectarrays with Rectangular Reflectors

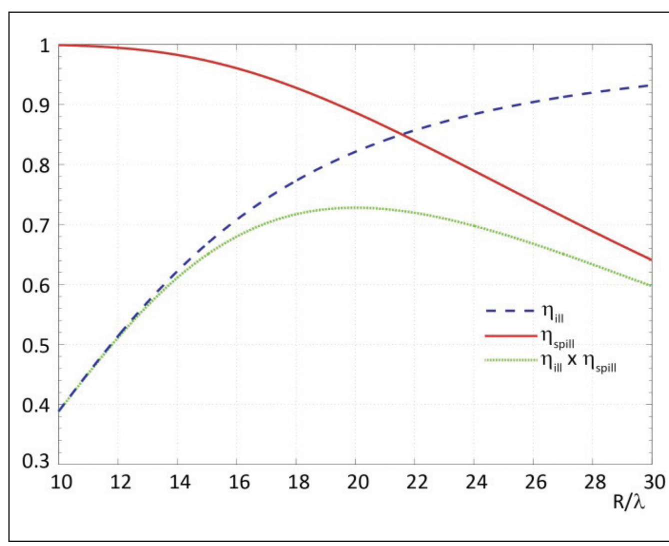
In this part of the paper efficiency coefficients η_{ill} and η_{spill}, as well as their product, were calculated and plotted against distance R, for different parameters a/b, q_E and q_H

Example 1

Let us consider a rectangular reflectarray, the reflector of which consists of 1089 elements, regularly placed in N=33 rows, with M=33 elements is each row. Moreover, let us assume that other antenna parameters are as follows: $d_x = d_y = 0.6\lambda$, $q_E = q_H = 10$. Such parameters mean that the 3dB beamwidths of the feed pattern are $\theta_E^{3dB} = \theta_H^{3dB} = 30^o$. Using the method described in paragraphs 2 and 3, efficiency coefficients η_{ill}, η_{spill} and $\eta_{ill} \chi \eta_{spill}$ were calculated as a function of the distance R. The results are illustrated using figure 4a.



(a)



(b)

Figure 4 • Spillover efficiency (solid red line), illumination efficiency (dashed blue line) and their product (dotted green line) for the rectangular reflectarray, Example 1 (a). Corresponding efficiencies for a circular reflectarray (b).

Efficiency coefficients calculated for a circular reflector reflectarray, the diameter of which is equal to the side length of the reflector from Example 1, are plotted in Fig 4b. For circular reflector closed form formulas from [8] were used. A short comparison of Fig. 4a and Fig. 4b leads to the conclusion that maximum efficiency (defined as the product of η_{ill} and η_{spill}) for rectangular reflectarray amounts to 0.71 and occurs for R = 22.5λ, whereas for circular reflectarray the efficiency is a little higher and goes up to 0.73 for R = 20λ.

It is also worth mentioning that for a given feed-reflector distance R, the illumination efficiency is always higher (closer to 1.0) for a circular reflectarray than for its rectangular equivalent. Such conclusion becomes evident when the shape of the reflectors and cross-section of the feed pattern are compared, showing that the corners of rectangular reflector are under-illuminated. On the contrary, spillover efficiency is always higher for rectangular reflectarray than for the circular one, assuming the same feed-reflector distance. This in turn may be explained by a bigger geometric area of the rectangular reflector, which has better "shielding" properties thanks to its corners.

Despite of these discrepancies, closed form formulas derived for circular reflectarrays may be used for square reflectarrays, at initial steps of design, provided that feed parameters are the same for circular and square reflectarray and the diameter of the circular reflector is equal to the side length of the square reflector. Under such circumstances the maximum efficiency of square reflectarray, calculated using closed form formulas (derived for circular reflectarray) is only a few percent higher (up to 5%) than the maximum efficiency calculated using the method from paragraphs 1 and 2.

Example 2

Let us now consider a reflectarray antenna with its reflector parameters unchanged from Example 1, but with a feed pattern described by $q_E = q_H = 3$, which means that its 3dB beamwidths are $\theta_E^{3dB} = \theta_H^{3dB} = 54^o$. Again, the presented formulas were utilized in order to calculate reflectarray efficiencies. The results are gathered in Figure 5.

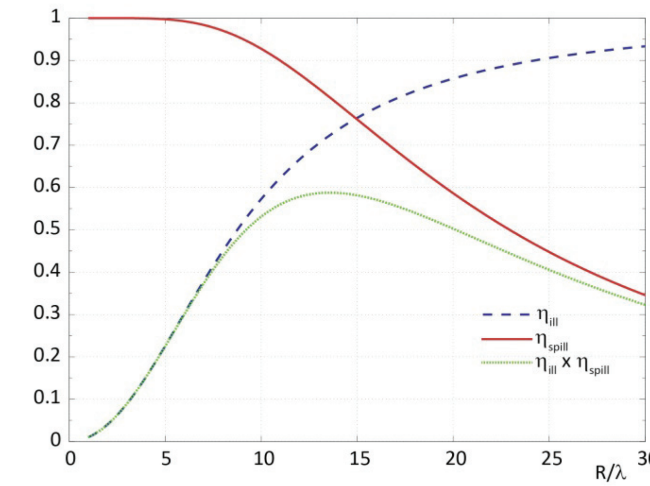
Similarly to Example 1, efficiency coefficients calculated for a circular reflector reflectarray, the diameter of which is equal to the side length of the reflector from Example 2, are plotted in Fig 5b. This time the maximum efficiency amounts to 0.59 for R = 13.5λ in case of a rectangular reflectarray. Meanwhile, for a circular reflectarray, the maximum efficiency is 0.60 for R = 12λ. Other conclusions are similar to those formulated in Example 1.

Example 3

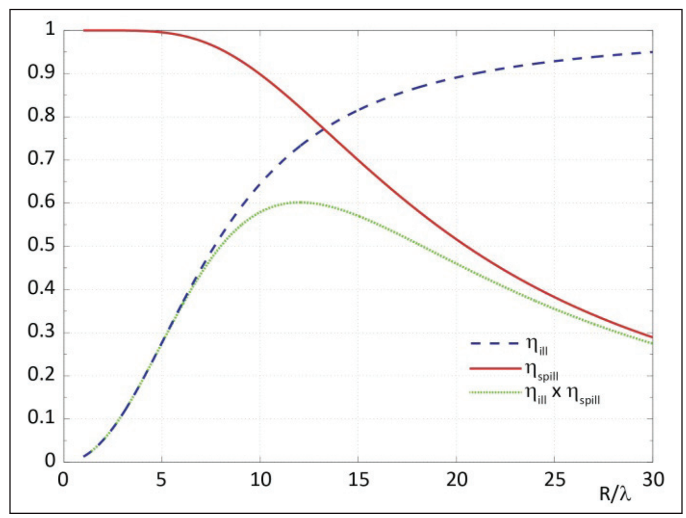
In the last example the formulas were used to calculate efficiencies for a rectangular (non-square) reflectarray. Let us assume that the antenna reflector consists of 561 reflectarray elements organized in N = 33 rows, with M = 17 elements is each row. Other reflectarray parameters are as follows: $d_x = d_y = 0.6\lambda$, $q_E = 10, q_H = 5$. The 3dB beamwidths of the feed pattern are $\theta_E^{3dB} = 30^o$ and $\theta_H^{3dB} = 42^o$. Calculated efficiencies plotted versus distance R are illustrated in Fig. 6

Maximal efficiency, again defined as $\eta_{spill} \chi \eta_{spill}$, amounts to 0.65, for R = 13.2λ. It is worth mentioning that in the present example efficiencies cannot be evaluated using closed form formulas (similar to Example 1 and 2), because the shape of the reflector is rectangular and feed pattern beamwidths are different in both principal planes.

For comparison, a rectangular reflectarray was analyzed with different feed pattern, shape of which is defined by $q_E = 5$, $q_H = 10$. Efficiencies calculated for this

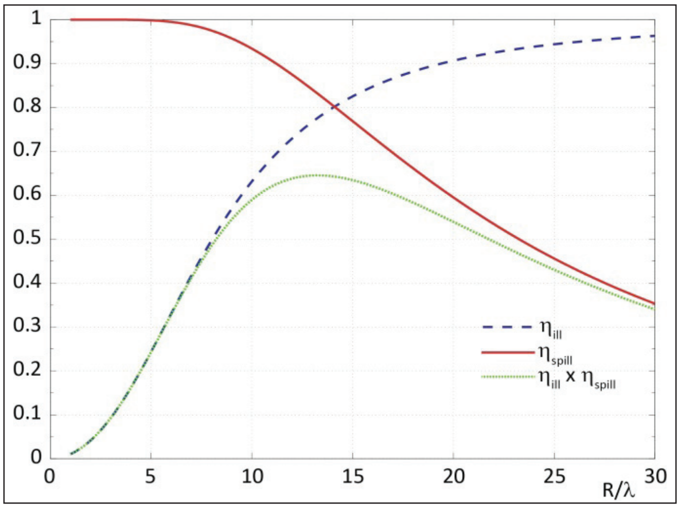


(a)

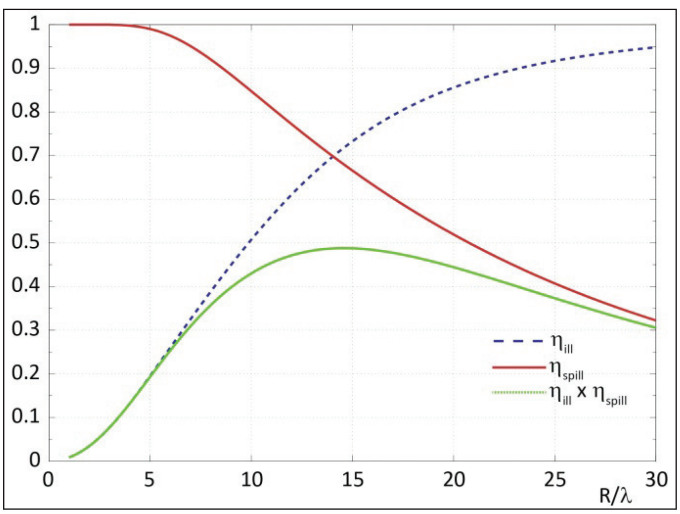


(b)

Figure 5 • Spillover efficiency (solid red line), illumination efficiency (dashed blue line) and their product (dotted green line) for the rectangular reflectivearray, Example 2 (a). Corresponding efficiencies for a circular reflectarray (b).



(a)



(b)

Figure 6 • Spillover efficiency (solid red line), illumination efficiency (dashed blue line) and their product (dotted green line) for the rectangular reflective array, Example 2 (a). Corresponding efficiencies for a reflectivearray with feed pattern parameters: $q_E = 5$, $q_H = 10$ (b).

reflectarray, plotted versus distance R are illustrated in Fig. 6b. Maximal efficiency, in this situation, amounts to 0.49 for $R = 14.5\lambda$. Comparison of results from Fig. 6a and 6b shows the importance of the right choice of reflectarray parameters. Choosing the wrong feed pattern parameters and distance R values for a given geometry of the reflector, leads to significantly lower global efficiency levels. By calculating illumination and spillover efficiencies with the presented method, optimal reflectarray parameters can be determined.

Conclusion

Formulas for determination of illumination and spillover efficiencies of rectangular reflectarrays have been presented along with a numerical method of their calculation. Appropriate determination of these parameters makes possible the choice of mutual position between the feed and reflector and feed pattern beamwidths, resulting in optimal global antenna efficiency. As an example, the formulas were used to calculate illumination and spillover efficiencies versus feed-reflector distance for several reflectarrays. The results were compared with corresponding results calculated using closed form formulas, known in literature and derived for circular reflectarrays.

Such formulas may also be used for square reflectarrays, provided that their feeds have the same parameters and their size is appropriate (as described in Example 1). In this case the difference in calculated global efficiency, using closed form formulas and the rigorous method presented herein, is in the order of a few percent. However, for rectangular reflectarrays closed form formulas do not exist, that is why the calculation method presented in this paper becomes useful. Apart from graphs presented in Examples 1,2 and 3, showing $\eta_{ill}(R)$ and $\eta_{spill}(R)$, different graphs can be plotted, that may help the designer choose correct reflectarray parameters. These graphs include illumination and spillover efficiencies $\eta_{ill,\ spill}(q_E)$, $\eta_{ill,\ spill}(q_H)$, $\eta_{ill,\ spill}(a)$, $\eta_{ill,\ spill}(b)$.

Appendix 1

Double integral calculation - Simpson method of integration

Double integral defined in a rectangular area D_{ij} can be written as follows [14]

$$\iint_{D_{ij}} f(x,y)dxdy = \int_{x_i-h}^{x_i+h} dx \int_{y_j-k}^{y_j+k} f(x,y)dy \qquad (17)$$

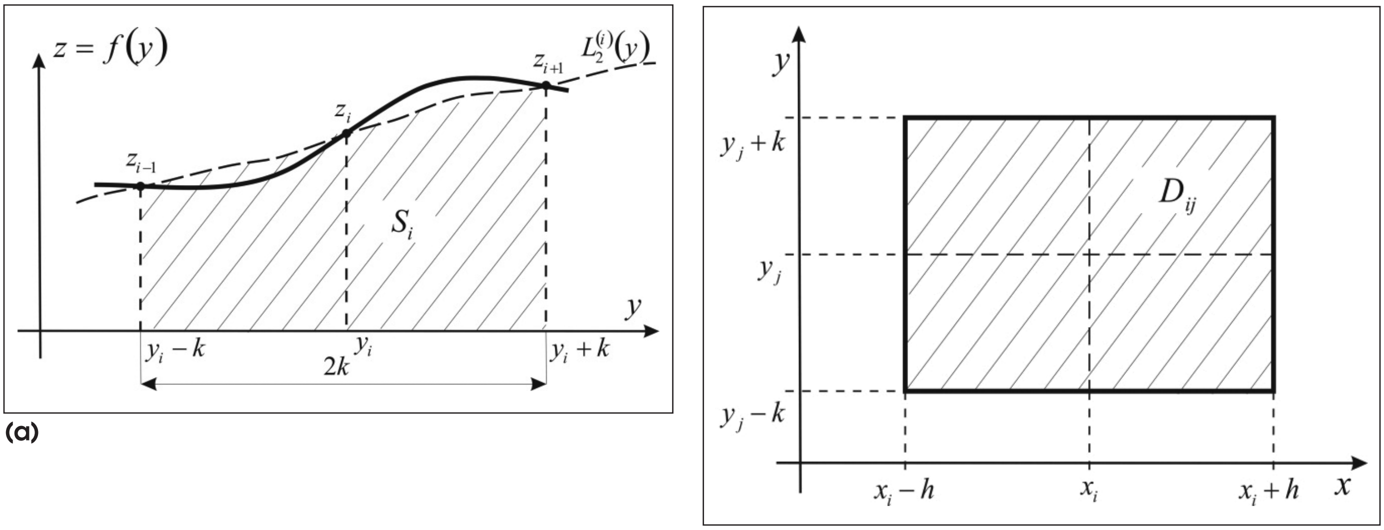
where the integrand in the second integral can be treated as a function of y dependent on parameter x. Single integral of function $f(y)$, dependent on parameter x, can be calculated using the quadrature Simpson formula, see Fig. 7a

$$S_i = \int_{y_{i-1}}^{y_{i+1}} L_2^{(i)}(y)dy = \frac{k}{3}\left(z_{i-1} + 4z_i + z_{i+1}\right) \qquad (18)$$

where $L_2^{(i)}$ is Lagrange polynomial of second order, which interpolates the integrand $f(y)$ [14].

Substituting (18) to expression (17), describing double integral gives:

$$\iint_{D_{ij}} f(x,y)dxdy = \int_{x_i-h}^{x_i+h} dx \cdot \frac{k}{3}\left[f(x,y_j-k) + 4f(x,y_j) + f(x,y_j+k)\right] =$$
$$= \frac{k}{3}\int_{x_i-h}^{x_i+h} f(x,y_j-k)dx + 4\cdot\frac{k}{3}\int_{x_i-h}^{x_i+h} f(x,y_j)dx + \frac{k}{3}\int_{x_i-h}^{x_i+h} f(x,y_j+k)dx \qquad (19)$$



(b)

Figure 7 • Integration areas in the Simpson method.

Using the quadrature Simpson formula again, results in an expression called the Simpson cubature formula or the Simpson type mechanic cubature [14][15]. Finally, the value of double integral (15) over the rectangular area Dij can be written as [14][15]:

$$\iint_{D_{ij}} f(x,y)dxdy = \frac{kh}{9}\left(A_{ij} + 4B_{ij} + 16C_{ij}\right)$$

where

$$A_{ij} = f(x_i - h, y_j - k) + f(x_i + h, y_j - k) + f(x_i - h, y_j + k) + f(x_i + h, y_j + k)$$
$$B_{ij} = f(x_i, y_j - k) + f(x_i, y_j + k) + f(x_i - h, y_j) + f(x_i + h, y_j)$$
$$C_{ij} = f(x_i, y_j)$$

If the integration area D is large, it is divided into $n \times m$ equal, sufficiently small sub–areas D_{ij}, with $i = 1, 2, 3, ..., n$, $j = 1, 2, 3, ..., m$, and their sides parallel to x and y axes. The values of the integral are calculated over every sub–areas D_{ij} and then summed to give the value of the integral over the whole integration area D.

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Michał Żebrowski received his M.S. degree in Electronic Engineering from Warsaw University of Technology (in Polish: Politechnika Warszawska), Warsaw, Poland, in 2007. He has been involved with Microwave Devices Dept. of the Bumar Elektronia SA and Telecommunications Research Institute, Warsaw, Poland, since 2007. His current research interests include antenna array systems, especially reflectarray antennas and passive microwave components.

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